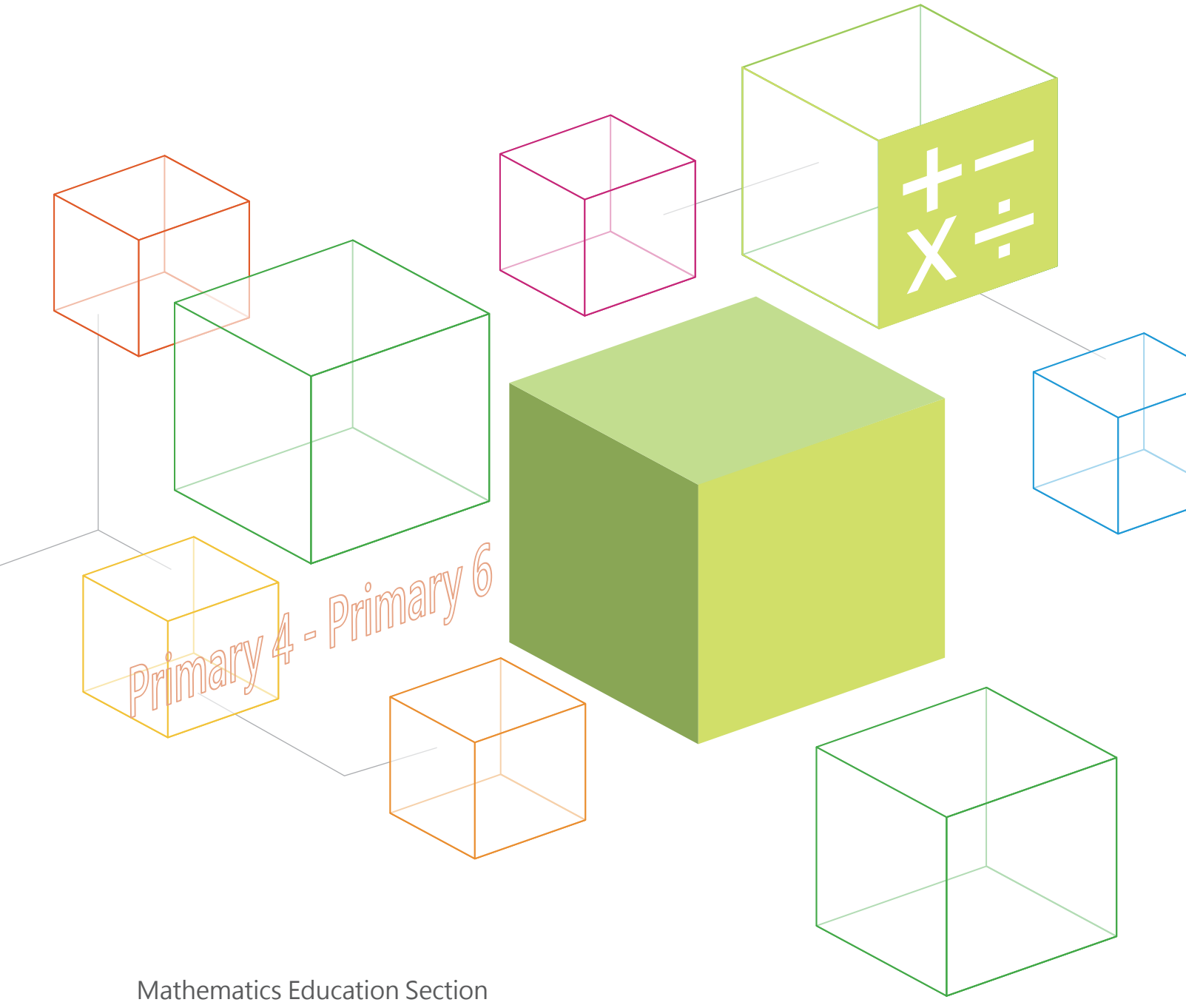
Explanatory Notes to Primary Mathematics Curriculum

(Key Stage 2)



Mathematics Education Section
Education Bureau
2020 (with updates in Nov 2026)

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Foreword

To keep abreast of the ongoing renewal of school curriculum at primary and secondary levels, the revised *Mathematics Education Key Learning Area Curriculum Guide (Primary 1 - Secondary 6)* (2017) and its supplements setting out the learning content at each key stage have been prepared by the Curriculum Development Council and released in late 2017. Among these documents, the *Supplement to Mathematics Education Key Learning Area Curriculum Guide: Learning Content of Primary Mathematics (2017)* (hereafter referred to as "*Supplement*") aims at elucidating in detail the learning targets and content of the revised primary Mathematics curriculum.

In the *Supplement*, the Learning Objectives of the primary Mathematics curriculum are grouped under different Learning Units in the form of a table. The notes in the "Remarks" column of the table provide supplementary information about the Learning Objectives.

The explanatory notes in this booklet aim at further explicating:

1. the requirements of the Learning Objectives of primary Mathematics curriculum;
2. the strategies suggested for the teaching of primary Mathematics curriculum;
3. the connections and structures among different Learning Units of primary Mathematics curriculum; and
4. the curriculum articulation between the primary Mathematics and the junior secondary Mathematics.

Teachers may refer to the "Remarks" column and the suggested lesson time of each Learning Unit in the *Supplement*, with the explanatory notes in this booklet being a supplementary reference, for planning the breadth and depth of treatment in learning and teaching. Teachers are advised to teach the content of the primary Mathematics as a connected body of mathematical knowledge and develop in students the capability for using mathematics to solve problems, reason and communicate. Furthermore, it should be noted that the ordering of the Learning Units and Learning Objectives in the *Supplement* does not represent a prescribed sequence of learning and teaching. Teachers may arrange the learning content in any logical sequences which take account of the needs of their students.

Comments and suggestions on this booklet are most welcomed. They may be sent to:

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Learning Unit	Learning Objective	Time
Number Strand		
4N1 Multiplication (II)	1. perform multiplication 2. perform multiplication by using the commutative and associative properties of multiplication 3. solve problems	6

Explanatory Notes:

In Primary Three, students have learnt how to perform the multiplication of a 1-digit number and a 2-digit number and that of a 1-digit number and a 3-digit number. In this learning unit, multiplication will be extended to:

- 2-digit number × 2-digit number
- 3-digit number × 2-digit number (or 2-digit number × 3-digit number)

Multiplication with carrying is required.

The requirements on the learning of multiplication at Key Stage 2 is similar to that at Key Stage 1. Students are required to both master the steps of performing multiplication in column form and understand the principle behind. Teachers should let students know that no matter it is the multiplication of a 2-digit number and a 2-digit number, or that of a 2-digit number and a 3-digit number, or that of a 2-digit number and a multi-digit number, the principle of operations remains the same.

Students have learnt the commutative property and the associative property of multiplication in Primary Two and Primary Three respectively. In this learning unit, teachers may design some concrete examples to facilitate students understand that flexible use of these two properties can speed up some calculations, so as to raise their interest in learning mathematics. For example,

$25 \times 53 \times 4$

$= 53 \times 25 \times 4$

$= 53 \times (25 \times 4)$

$= 53 \times 100$

$= 5300$

Although teachers may use the terms “associative property” and “commutative property” in their explications, the curriculum does not require students to use these two terms.

After students have mastered the skills of performing multiplication, teachers should guide students to estimate the results of calculations and let them understand the advantages of estimation, so as to enhance students’ abilities in solving related problems.

Learning Unit	Learning Objective	Time
Number Strand		
4N2 Division (II)	1. perform division 2. recognise the concept of divisibility 3. solve problems	6

Explanatory Notes:

At Key Stage 1, the divisors in division are 1-digit numbers. In this learning unit, the division will be extended to:

- 2-digit number ÷ 2-digit number
- 3-digit number ÷ 2-digit number

Division with borrowing and division involving remainder are required.

The requirements on the learning of division at Key Stage 2 is similar to that at Key Stage 1. Students are required to both master the steps of performing division in column form and understand the principle behind. Teachers should let students know that no matter it is the division of a 2-digit number divided by a 2-digit number, or that of a 3-digit number divided by a 2-digit number, or that of a multi-digit number divided by a 2-digit number, the principle of operations remains the same.

Students are required to recognise that 0, 1, 2, 3, 4, … are whole numbers* and the concept of divisibility. For example, 24 is divisible by 3 because 24 ÷ 3 = 8. At the primary level, students need to recognise the divisibility tests for 2, 3, 5 and 10, but their proofs are not required by the curriculum. The divisibility tests for 4, 6, 8 and 9 are the learning content at junior secondary level. Learning divisibility helps students to enhance their computation skills. For example, when performing multiplication and division of fractions, students can find the common factors of some numbers in a quicker way.

At Key Stage 1, students determined whether a whole number is odd or even by its units digit. In this learning unit, students are required to further recognise the concepts of odd and even numbers through the divisibility by 2.

* Directed numbers belong to the learning content of the junior secondary Mathematics curriculum. In the primary Mathematics curriculum, "whole numbers" refers to non-negative integers. Teachers need not introduce the term "positive integers" to primary students.

After students have mastered the skills of performing division, teachers should guide students to estimate the results of calculations and let them understand the advantages of estimation, so as to enhance students' abilities in solving related problems.

Examples of vocabularies to be learnt: whole number, divisible, even number, odd number, etc.

Learning Unit	Learning Objective	Time
Number Strand		
4N3 Multiples and factors	1. understand the concept of multiples 2. understand the concept of factors 3. understand the relation between factors and multiples 4. recognise the concepts of prime numbers and composite numbers	8

Explanatory Notes: (*In this learning unit, numbers refer to positive integers only*.)

Teachers should let students understand the concept of multiples through the concept of multiplication. For example, as 5×1=5 (or 1×5=5), 5×2=10, 5×3=15, 5×4=20, the numbers 5, 10, 15, 20 are the multiples of 5. But students are not required to recognise that 0 is a multiple of 5.

Teachers should let students understand the concept of factors through the concept of divisibility. For example, as 20 is divisible by 1 and 5, but not divisible by 3, 1 and 5 are the factors of 20 but 3 is not.

Students should find the multiples of a number by multiplication, such as the first multiple, the second multiple, the third multiple… of the number. They should find all the factors of a number. They should determine whether a given number is a multiple or a factor of another number. Teachers should also let students conclude that 1 is the factor of any numbers while all numbers are multiples of 1. But the concepts of the multiples and factors of 0 are not required by the curriculum.

Students are required to understand that the concepts of multiples and factors are closely related. For example, 8 is a multiple of 2 and 2 is a factor of 8. Teachers should let students grasp the relation between factors and multiples through the concept of divisibility. For example, as 51 is divisible by 3, 3 is a factor of 51and 51 is a multiple of 3.

Students are required to recognise that when a number has only two factors (i.e. 1 and that number), it is a prime number. 1 has only one factor while a composite number has more than two factors. Teachers should explain to students that 1 is neither a prime number nor a

composite number.

Students should use the concept of divisibility to determine whether a given number within 100 is a prime number. They should find all the prime numbers within 1 to 100 by the Sieve of Eratosthenes.

Examples of vocabularies to be learnt: multiple, factor, prime number, composite number, Sieve of Eratosthenes, etc.

Learning Unit	Learning Objective	Time
Number Strand		
4N4 Common multiples and common factors	1. understand the concepts of common multiples and common factors 2. understand the concepts of the least common multiple and the highest common factor 3. find the least common multiple and the highest common factor of two numbers by listing their multiples and factors 4. find the least common multiple and the highest common factor of two numbers by using short division	7.5

Explanatory Notes: (*In this learning unit, numbers refer to positive integers only*.)

In this learning unit, students are required to understand the concepts of common factors, common multiples, the highest common factor, and the least common multiple of two numbers. They are required to recognise the short forms "H.C.F." and "L.C.M." for the highest common factor and the least common multiple respectively.

Students should find the L.C.M. and the H.C.F. of two numbers by listing their multiples and factors and by short division. The curriculum does not require students to understand the reasons why short division works or the proof for it. Yet, teachers may consider to use examples to illustrate the principle behind short division according to students' abilities.

During the discussion of finding the L.C.M. of two numbers by listing their multiples, teachers should let students discover the relation between L.C.M. and other common multiples through observation. For example, L.C.M. of 4 and 6 is 12, and their third common multiple is 36, which is 3 times the L.C.M. However, the curriculum does not require students to recognise the relation between the H.C.F. of two numbers and their other common factors, nor to recognise that the product of two numbers equals the product of their H.C.F. and L.C.M.

Teachers may help students to explore if the greater number of two numbers is the multiple of the smaller number, then the H.C.F. is the smaller number and the L.C.M. is the greater number. For example, the H.C.F. of 2 and 4 is 2. The L.C.M. of 2 and 4 is 4. Teachers may also consider to discuss with students the case when 1 is the only common factor of two numbers.

Examples of vocabularies to be learnt: common multiple, common factor, the least common multiple, the highest common factor, H.C.F., L.C.M., listing, short division, etc.

Learning Unit	Learning Objective	Time
Number Strand		
4N5 Four arithmetic operations (II)	1. recognise the distributive property of multiplication 2. perform mixed operations of not more than four numbers 3. perform mixed arithmetic operations of not more than five numbers 4. solve problems	8

Explanatory Notes:

Students are required to recognise the distributive property of multiplication and its applications. Teachers should let students understand that flexible use of the distributive property of multiplication can speed up some calculations, so as to raise students' interest in learning mathematics. For example,

$$\begin{aligned} & 23\times17+23\times13 \\ = \ & 23\times(17+13) \\ = \ & 23\times30 \\ = \ & 690 \end{aligned}$$

$$\begin{aligned} & 4\times(25+8) \\ = \ & 4\times25+4\times8 \\ = \ & 100+32 \\ = \ & 132 \end{aligned}$$

Although teachers may use the term "distributive property" in their explication, the curriculum does not require students to use this term.

Students should perform the mixed operations of not more than four numbers, the mixed operations include:

- mixed operations of division and addition
- mixed operations of division and subtraction
- mixed operations of division and multiplication

Students are required to recognise the order of performing mixed operations, including the mixed operations involving brackets. For students with adequate abilities, teachers may discuss with them through concrete examples whether division possess the distributive property.

Students should also be able to perform mixed arithmetic operations of not more than five numbers. Through concrete examples involving at least two different operations of addition, subtraction, multiplication and division, teachers should let students know how to follow the conventional order of mixed operations in performing calculations with or without brackets. And it is to let students understand that the flexible use of the properties of addition and multiplication can speed up some calculations. For example:

$$
\begin{aligned}
& 48\times6+42 \\
= \ & 48\times(5+1)+42 \\
= \ & 240+48+42 \\
= \ & 240+90 \\
= \ & 330
\end{aligned}
$$

At the primary level, mixed operations may involve more than one pair of brackets, such as $(2+3) \times (5+3)$, but operations involving multiple levels of brackets, such as $(4-(2-1))\times3$, are not required.

Students should able to solve problems involving direct proportion by the unitary method. For example, “72 dollars can buy six pencils. How much do eight pencils cost?” or “72 dollars can buy six pencils. How many pencils can be bought with $120?”. Teachers can guide students to find the price of a pen first, and use the result to find the required cost or number of pencils. The curriculum does not require students to use the term “direct proportion”. Teachers should encourage students to solve complicated problems by parts.

After students have mastered the skills of performing mixed arithmetic operations, teachers should guide students to estimate the results of calculations and let them understand the advantages of estimation, so as to enhance students’ abilities in solving related problems.

Learning Unit	Learning Objective	Time
Number Strand		
4N6 Fractions (II)	1. recognise the concepts of proper fractions, improper fractions and mixed numbers 2. perform the interconversions between improper fractions and mixed numbers 3. recognise the concepts of expanding fractions and reducing fractions 4. compare the magnitude of fractions with the same denominators 5. perform addition and subtraction of not more than three fractions with the same denominators 6. perform mixed operations of addition and subtraction of three fractions with the same denominators 7. solve problems	9

Explanatory Notes:

Students are required to recognise the concepts of proper fractions and improper fractions. They are required to recognise that when the numerator of an improper fraction is not a multiple of its denominator, the fraction can be written as a combination of a whole number and a proper fraction, which is called a "mixed number". Thus, students are required to recognise that a mixed number is the sum of a whole number and a proper fraction. Moreover, students should perform the interconversions between improper fractions and mixed numbers (or whole numbers), e.g. the interconversion between $\frac{6}{2}$ and 3.

In Primary Three, students have learnt the concept of equivalent fractions not greater than 1. In this learning unit, teachers should let students discover that when the numerator and denominator of a fraction are multiplied or divided by the same non-zero whole number, the value of the fraction remains unchanged, so as to recognise the concepts of expanding fractions and reducing fraction. Students are also required to recognise "fractions in the lowest terms" are those fractions with 1 as the H.C.F. of their numerators and denominators, such as, $\frac{2}{3}$, $\frac{5}{2}$,

$2\frac{1}{2}$.

Students should compare the magnitudes of fractions with the same denominator. They should also be able to compare the magnitudes of fractions and whole numbers, such as comparing the magnitudes of $1\frac{1}{4}$, $\frac{7}{4}$, $\frac{3}{4}$ and 2.

Teachers should let students understand the concepts of addition and subtraction of fractions through the concepts of addition and subtraction of whole numbers. Students should perform addition and subtraction of not more than three fractions and mixed operations of addition and subtraction of three fractions with the same denominator. Performing addition, subtraction and their mixed operations for fractions and whole numbers are required, such as $5 + \frac{1}{3}$, $3 - 1\frac{1}{4}$ and $3 - 1\frac{1}{4} + \frac{3}{4}$. If the result of calculations is a fraction, it should be reduced to the lowest terms. If it is greater than 1, it may be expressed as a fraction in the lowest terms, either as a mixed number or an improper fraction, such as $\frac{5}{2}$ and $2\frac{1}{2}$.

After students have mastered the skills of performing addition, subtraction and mixed operations of addition and subtraction of fractions with the same denominator, teachers should guide students to estimate the results of calculations and let them understand the advantages of estimation, so as to enhance students' abilities in solving related problems.

Examples of vocabularies to be learnt: proper fraction, improper fraction, mixed number, expanding fraction, reducing fraction, fraction in the lowest term, etc.

Learning Unit	Learning Objective	Time
Number Strand		
4N7 Decimals (I)	1. recognise the concept of decimals 2. recognise the concepts of tenth, hundredth, thousandth and ten thousandth places 3. compare the magnitude of decimals 4. recognise the daily life applications of decimals	3

Explanatory Notes:

Teachers should let students recognise that the decimal point in a decimal divides the decimal into the integer part and the decimal part. Teachers should let students recognise the concepts of tenths, hundredths, thousandths and ten thousandths places of decimals through the concept of fractions. Students are required to recognise the meaning of the numerals in tenths, hundredths, thousandths and ten thousandths places, so as to understand that decimals and fractions are two forms of expressing numbers and the relation between them.

Students are required to recognise that when reading a decimal, the numerals in the decimal part are read one by one from left to right, such as 30.102 being read as “thirty point one zero two". They are also required to recognise that either adding "0" to the end of the decimal or deleting "0" from the end of the decimal does not change the value of the decimal.

Students are required to recognise the classification of decimals. For example, both 0.5 and 24.7 are decimals of 1 decimal place where as 0.15 and 4.07 are decimals of 2 decimal places. Similarly, 0.135 and 10.315 are decimals of 3 decimal places where as 0.1224 and 20.1224 are decimals of 4 decimal places.

Students should compare the magnitudes of decimals, and compare the magnitudes of whole numbers, fractions (with denominators being factors of 100) and decimals, e.g. comparing the magnitudes of 12.35, 12.205, 120, and comparing the magnitudes of 12.05, $12\frac{1}{4}$, 120.

Students are required to recognise the daily life applications of decimals, e.g. the representation of amounts of money, length, capacity, etc. Students should interconvert between units in measurement. For example:

1.234L = 1234mL ;

23 dollars 50 cents = 23.5dollars .

Complicated interconversions between units are not required. For example:

0.4 km = 40000 cm .

Interconversion between the units of time is dealt with in Learning Unit 6M4 “Speed”.

Examples of vocabularies to be learnt: decimal point, tenths place, hundredths place, thousandths place, ten thousandths place, etc.

Learning Unit	Learning Objective	Time
Number Strand		
4N8 Decimals (II)	1. perform addition and subtraction of not more than three numbers 2. perform mixed operations of addition and subtraction of three numbers 3. solve problems	4.5

Explanatory Notes:

Teachers should let students understand the concepts of addition and subtraction of decimals through the concepts of addition and subtraction of whole numbers.

Students should perform addition, subtraction and mixed operations of whole numbers and decimals of at most 2 decimal places. Teachers should let students recognise that the steps of performing addition and subtraction of decimals are similar to those of addition and subtraction of whole numbers. They are required to recognise that the commutative and associative properties of addition for whole numbers also hold for decimals. Teachers should teach students to write the whole numbers as decimals when necessary, such as 4 = 4.00.

The numbers of digits involved in the addition and subtraction of decimals should not exceed that required for the addition and subtraction of whole numbers in Learning Unit 3N4 “Four arithmetic operations (I)”, i.e. four digits. For example, the following addition and subtraction of decimals are not required:

- 1.2345 + 5.6
- 123.4 + 56.78
- 1234 – 5.6

After students have mastered the skills of performing addition, subtraction and mixed operations of addition and subtraction of decimals, teachers should guide students to estimate the results of calculations and let them understand the advantages of estimation, so as to enhance students’ abilities in solving related problems.

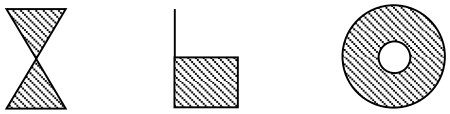
Learning Unit	Learning Objective	Time
Measures Strand		
4M1 Perimeter (I)	1. recognise the concept of perimeter 2. measure and compare the perimeters of 2-D shapes 3. recognise and use the formulae for finding the perimeters of squares and rectangles	6.5

Explanatory Notes:

In this learning unit, students have to recognise the concept of perimeter. Teachers may guide students to recognise the concept by using different 2-D shapes as examples and non-examples, such as:

 (example)  (non-example)

Students have learnt the concept of length and different units of length at Key Stage 1. Teachers may let students measure and compare the perimeters of different simple 2-D shapes (including regular and irregular shapes) for consolidating their recognition on the connection between length and perimeter. However, students are not required to find the perimeters of non-simple 2-D shapes such as:



After students have acquired the experience of making measurements, teachers should teach them how to estimate the results of measurements and encourage them to estimate the result before measuring.

In Learning Objective 3, teachers may guide students to recognise the formulae for finding the perimeters of squares and rectangles through some inquiry activities, such as measuring and recording the lengths of sides and perimeters of some squares and rectangles. Students are required to use the formulae for finding the perimeters of squares and rectangles to solve problems.

Students are required to find the perimeters of 2-D shapes formed by squares and rectangles.

Although operations of the related problems may involve more than five numbers, teachers should avoid selecting and designing problems with complicated calculations for students to solve.

Examples of vocabularies to be learnt: perimeter, length of a side, etc

Learning Unit	Learning Objective	Time
Measures Strand		
4M2 Area (I)	1. recognise the concept of area 2. compare intuitively the areas of 2-D shapes 3. compare directly the areas of 2-D shapes 4. compare the areas of 2-D shapes in improvised units 5. recognise square centimetre (cm^2) and square metre (m^2) 6. measure and compare the areas of 2-D shapes in square centimetre and square metre 7. recognise and use the formulae for areas of squares and rectangles	7

Explanatory Notes:

In this learning unit, students have to recognise the concept of area. Teachers may design learning activities together with daily-life examples to introduce the concept, such as discussion on the sizes of a textbook cover and a desktop, etc. In Learning Objectives 2, students are required to compare the areas of 2-D shapes intuitively. Teachers should select two 2-D shapes which are significantly different in area to enable students to compare their areas by observation. In Learning Objective 3, students are required to compare the areas of 2-D shapes directly through overlapping the shapes. Teachers may discuss with students the points to note when comparing the areas of 2-D shapes directly. For example, a 2-D shape has to cover another 2-D shape completely.

In this learning unit, students are required to compare the areas of 2-D shapes in improvised units. Students are also required to indirectly compare the areas of 2-D shapes by the comparison with a third 2-D shape. For examples:

- If A and B are of equal area and the area of B is smaller than that of C, the area of A is smaller than that of C.
- If the area of A is greater than that of B and the area of B is greater than that of C, the area of A is greater than that of C.
- If the area of A is smaller than that of B and also smaller than that of C, the area of A is the smallest.

Teachers may discuss with students the points to note when using the improvised units. Students are required to choose appropriate improvised units for taking measurements based on actual circumstances. Teachers may guide students to find the areas of regular and irregular shapes using squared paper and understand the need for using standard units through activities.

After introducing the units of area square centimetre (cm^2) and square metre (m^2), teachers may conduct various activities to enhance students' recognition on square centimetre and square metre. Students are required to measure and compare the areas of 2-D shapes in square centimetre and square metre. However, interconversion between square centimetre and square metre is not required.

After students have acquired the experience of making measurements, teachers should teach them how to estimate the results of measurements and encourage them to estimate the result before measuring.

In Learning Objective 7, teachers may guide students to recognise the formulae for areas of squares and rectangles through some inquiry activities, such as counting the number of squares covered by a rectangle. Students are required to use the formulae for areas of squares and rectangles to solve problems. Finding the length of side of a rectangle from its area may be involved.

Students are required to find the areas of 2-D shapes formed by squares and rectangles. Although operations of the related problems may involve more than five numbers, teachers should avoid selecting and designing problems with complicated calculations for students to solve.

Examples of vocabularies to be learnt: area, square centimetre (cm^2), square metre (m^2), etc.

Learning Unit	Learning Objective	Time
Shape and Space Strand		
4S1 Quadrilaterals (III)	1. recognise the concept and properties of rhombuses 2. draw and make rhombuses 3. recognise the relations between different types of quadrilaterals	8

Explanatory Notes:

Students are required to recognise that a quadrilateral with four equal sides is a rhombus and its opposite sides are parallel. They should identify rhombus according to its properties.

Teachers may provide students with different kinds of paper, such as dot grid paper, grid paper and blank paper, etc. to draw rhombuses of different shapes and sizes. Teachers may also provide students with different materials such as cotton strings, straws and geometric strips, etc. to make rhombuses of different shapes and sizes.

Students have recognised the concepts and properties of rectangles, squares and parallelograms and recognised that squares and rectangles are parallelograms at Key Stage 1. In this learning unit, students are required to further recognise the following inclusion relations among the quadrilaterals through the comparison of their properties:

- all squares are rectangles
- all squares, rectangles and rhombuses are parallelogram
- all squares are rhombuses

Teachers may use diagrams such as Venn diagrams or tree diagrams to help students recognise the inclusion relations between different types of quadrilaterals required by the curriculum, but students are not required to construct Venn diagrams and tree diagrams. They are also not required to use the term “inclusion relation”.

Examples of vocabularies to be learnt: rhombus, etc.

Learning Unit	Learning Objective	Time
Shape and Space Strand		
4S2 Dissecting and forming shapes	1. dissect a polygon into smaller polygons 2. form a polygon by smaller polygons	3

Explanatory Notes:

Students are required to recognise that polygons are formed by connecting at least three line segments end to end on a plane, e.g. triangles, quadrilaterals, pentagons, etc.

In this learning unit, students are required to dissect a polygon into several smaller polygons, and form a polygon by putting several smaller polygons together.

After dissecting a polygon, students should determine the properties of the smaller polygons according to the properties of the original polygon. For example, dissecting a rectangle diagonally will get two right-angled triangles; while dissecting a square diagonally will give two isosceles right-angled triangles. Students are not required to use the term “diagonal”.

Students are required to determine some of the properties of the polygon formed by several polygons according to the properties of the original polygons. For example, two right-angled trapeziums of the same size and shape may form a rectangle.

Examples of vocabularies to be learnt: polygon, dissecting, forming, etc.

Learning Unit	Learning Objective	Time
Shape and Space Strand		
4S3 Directions and positions (III)	1. recognise the four directions: southeast, northeast, southwest and northwest 2. use the compass to measure directions	3.5

Explanatory Notes:

Students have learnt the four main directions east, south, west and north in Primary Two. In this learning unit, students are required to recognise the four directions "southeast", "northeast", "southwest" and "northwest", and their respective short forms "SE", "NE", "SW" and "NW". Students are also required to use the sentence "_____ is to the southeast/northeast/southwest/northwest of _____ " to describe the relative positions of objects.

Students are required to find out the above eight directions with a compass and recognise some simple applications of compass, e.g. using the directions indicated on a road map to choose paths. Instructions of turning left or right may be included, e.g. turn left by two right angles.

Examples of vocabularies to be learnt: southeast, northeast, southwest, northwest, etc.

Learning Unit	Learning Objective	Time
Data Handling Strand		
4D1 Bar charts (II)	1. recognise bar charts of greater frequency counts 2. interpret bar charts of greater frequency counts 3. recognise the concept of approximate values 4. construct bar charts of greater frequency counts	5

Explanatory Notes:

Students have learnt the bar charts using one-to-one, one-to-two and one-to-five representations in Primary Three. Teachers may make use of some statistical data with greater frequency counts, e.g. the total number of different types of books in the school library, to guide students to recognise the need for using bar charts of greater frequency counts.

In this learning unit, students are only required to interpret and construct bar charts using one-to-ten, one-to-fifty and one-to-hundred representations. Bar charts in horizontal and vertical forms are required.

For Learning Objective 3, students can round off data to fit the chosen representation and construct bar charts using the one-to-ten, one-to-fifty and one-to-hundred representations after recognising the concept of approximate values. Thus, teachers may make use of examples involving rounding off numbers to the nearest tens or hundreds when teaching this learning objective. For rounding off numbers to the nearest places other than tens or hundreds, it should be taught in Learning Unit 5N1 “Multi-digit numbers”.

Teachers may select some daily life examples or some topics that students are familiar with for them to construct bar charts of greater frequency counts, e.g. the favourite co-curricular activities for Primary Four students. Teachers should guide students to classify data, use frequency tables for recording data, choose the one-to-ten, one-to-fifty or one-to-hundred representations according to the magnitude of the data and round off data to fit the chosen representation. Teachers may remind students the points to note when constructing bar charts, e.g. if “others” is a category for classifying data, its quantity should better be a small proportion of the whole.

Apart from using paper and pencil, teachers may let students use IT to construct bar charts of greater frequency counts.

Examples of vocabularies to be learnt: approximate value, classify, rounding off, etc.

Learning Unit	Learning Objective	Time
Further Learning Unit		
4F1 Inquiry and investigation	Through various inquiry activities and mathematical modelling learning activities, construct and apply mathematical knowledge, further improve the ability to inquire, communicate, reason, conceptualise mathematical concepts and apply mathematics	10

Explanatory Notes:

This Learning Unit aims at providing students with more opportunities to engage in the activities that avail themselves of discovering and constructing knowledge, further improving their abilities to inquire, communicate, reason and conceptualise mathematical concepts when studying other Learning Units; as well as enhancing their abilities to apply mathematics to solve real-world problems through mathematical modelling activities. In other words, this is not an independent and isolated learning unit. The time is allocated for students to engage in learning activities from different learning units, for example, activities on enrichment topics, cross learning unit activities, and cross-KLA activities that based on mathematical topics. Among the learning activities, teachers should plan about 2–3 hours of mathematical modelling learning activities for students in each school year, enabling them to analyse problems and make decisions with mathematical thinking when encountering problems in different real-world contexts. and strengthening their ability to apply mathematics to solve STEAM and other real-world problems. Teachers may make good use of Mathematics lessons, lessons designated for project learning or cross-disciplinary learning, or other suitable lessons, together with other learning time, such as after-school or post-examination activity periods, to conduct mathematical modelling learning activities. These learning activities should be regarded as STEAM learning activities of Mathematics.

For details of the learning and teaching of mathematical modelling, please refer to pages 63 –72.

Learning Unit	Learning Objective	Time
Number Strand		
5N1 Multi-digit numbers	1. recognise the concept of multi-digit numbers 2. compare the magnitude of numbers 3. use rounding off to obtain approximate value of multi-digit numbers 4. estimate large quantities	3

Explanatory Notes:

The concept of multi-digit numbers can be viewed as an extension of the concept of five-digit numbers. Students are required to recognise multi-digit numbers by counting, reading and writing. For example, they should recognise 10 ten thousand equal to 1 hundred thousand, 10 hundred thousand equal to 1 million, 10 million equal to 1 ten million, 10 ten million equal to 1 hundred million, and the meaning of the numerals in the hundred thousands place, millions place, ten millions place and hundred millions place.

Students should perform counting onwards and backwards of multi-digit numbers and to determine whether a multi-digit number is an odd or even number by the test of divisibility of 2. Teachers should let students know that the method of comparing the magnitudes of two multi-digit numbers is similar to that of comparing numbers learnt at Key Stage 1.

In Primary Four, students have learnt the concept of approximation in Learning Unit 4D1 “Bar charts (II)”. In this learning unit, students should recognise to use rounding off to obtain approximate values of multi-digit numbers to the nearest thousands, ten thousands, hundred thousands, millions, ten millions or hundred millions, such as the approximate value of 123456789 equals to 123460000 to the nearest ten thousands.

Students should estimate large quantities, for example, number of words in a book and number of people attending large-scale events, etc. In addition to teaching students how to do estimation, teachers should also let students understand that different methods of estimation may have different degrees of accuracy.

Examples of vocabularies to be learnt: multi-digit number, hundred thousands place, millions place, ten millions place, hundred millions place, rounding off, etc.

Learning Unit	Learning Objective	Time
Number Strand		
5N2 Fractions (III)	1. compare the magnitude of not more than three fractions with different denominators 2. perform addition and subtraction of not more than three fractions with different denominators 3. perform mixed operations of addition and subtraction of three fractions with different denominators 4. solve problems	8.5

Explanatory Notes:

Students should compare the magnitudes of not more than three fractions with different denominators. Comparing the magnitudes of fractions and whole numbers is required, such as comparing the magnitudes of $1\frac{5}{12}$, $\frac{7}{6}$, $\frac{3}{4}$ and comparing the magnitudes of $1\frac{1}{15}$, $\frac{7}{6}$, 2.

Students should perform addition and subtraction of not more than three fractions with different denominators and perform mixed operations of addition and subtraction of three fractions with different denominators. Addition, subtraction and mixed operations of addition and subtraction of fractions and whole numbers are required, such as $3 - 1\frac{1}{15} + \frac{5}{6}$. Teachers should avoid requiring students to perform complicated comparisons and mixed operations of addition and subtraction of fractions with different denominators. When the operations and comparison involving three fractions with different denominators, all given denominators should not exceed 12.

Students are required to recognise that the different denominators should be firstly converted into the same denominator by expanding the fractions, and then perform the addition and subtraction of fractions with the same denominator. Teachers should let students recognise that this common denominator could be any common multiple of these different denominators. However, finding the L.C.M. of three numbers by listing method or short division is not required by the curriculum. Teachers may teach students to determine the common multiple to be used as the denominator by finding a common multiple of two numbers each time. For

example, when performing the operation of $1\frac{3}{8} + \frac{5}{6} - \frac{2}{9}$, students can use 24 as the same denominator for $1\frac{3}{8} + \frac{5}{6}$, as the least common multiple of 8 and 6 is 24, and use 72 as the same denominator for the subsequence subtraction as the least common multiple of 24 and 9 is 72. If the result of calculations is a fraction, it should be reduced to the lowest terms. If it is greater than 1, it may be expressed as a fraction in the lowest terms, either as a mixed number or an improper fraction.

After students have mastered the skills of performing addition, subtraction and mixed operations of addition and subtraction of fractions with different denominators, teachers should guide students to estimate the results of calculations and let them understand the advantages of estimation, so as to enhance students' abilities in solving related problems.

Examples of vocabularies to be learnt: fractions with different denominators, etc.

Learning Unit	Learning Objective	Time
Number Strand		
5N3 Fractions (IV)	1. perform multiplication of not more than three numbers 2. solve problems	8

Explanatory Notes:

Teachers should let students understand the concept of multiplication of fractions through the concept of multiplication of whole numbers.

Students should perform multiplication of not more than three fractions in which whole number can be included.

Teachers should avoid requiring students to perform complicated multiplication of fractions. Multiplication of three fractions should not involve more than one mixed number.

Teachers should let students revise the conversions of mixed numbers into improper fractions and the conversions of whole numbers into improper fractions with denominator 1, for example $4 = \frac{4}{1}$, to let students recognise the rules of multiplication of fraction. The result of calculations can be expressed as a mixed number or an improper fraction in the lowest terms. If the result of calculations is a fraction, it should be reduced to the lowest terms. If it is greater than 1, it may be expressed as a fraction in the lowest terms, either as a mixed number or an improper fraction.

After students have mastered the skills of performing multiplication of fractions with different denominators, teachers should guide students to estimate the results of calculations and let them understand the advantages of estimation, so as to enhance students' abilities in solving related problems.

Learning Unit	Learning Objective	Time
Number Strand		
5N4 Decimals (III)	1. perform multiplication of a number and 10, 100, 1000 2. perform multiplication of a number and 0.1, 0.01, 0.001 3. perform multiplication of two numbers 4. solve problems	7

Explanatory Notes:

Teachers should let students understand the concept of multiplication of decimals through the concepts of multiplication of whole numbers and multiplication of fractions.

Teachers should provide students with concrete examples to facilitate them to discover that multiplying a whole number or a decimal by 10, 100 and 1000, are equivalent to moving the decimal point to the right by one, two and three digits respectively; while multiplying a whole number or a decimal by 0.1, 0.01 and 0.001, are equivalent to moving the decimal point to the left by one, two, and three digits respectively.

Students should perform the following multiplication of decimals:

- a decimal × a whole number (or a whole number × a decimal)
- a decimal × a decimal

Except for the multiplications in Learning Objectives 1 and 2, the number of decimal places of the decimals involved in multiplications should be one or two. Besides, the numbers of digits involved in the multiplication of decimals should not exceed those involved in the multiplication of whole numbers as required in Leaning Unit 4N1 “Multiplication (II)” (i.e. not more than 3-digit × 2-digit). For example, the following multiplication of decimals are not required:

- 0.124 × 3.9
- 12.4 × 3.42
- 12.41 × 2.6

When performing the multiplication of decimals, students should determine the position of the decimal point of the product and round off the result of calculations to the nearest tenth or

hundredth when necessary. Students are required to use the symbol "≈" to indicate approximately equal.

After students have mastered the skills of performing multiplication of decimals, teachers should guide students to estimate the results of calculations and let them understand the advantages of estimation, so as to enhance students' abilities in solving related problems.

Learning Unit	Learning Objective	Time
Number Strand		
5N5 Fractions (V)	1. further recognise the concept of fractions 2. perform division of not more than three numbers 3. perform mixed arithmetic operations of three numbers 4. solve problems	9.5

Explanatory Notes:

Teachers should let students understand the concept of division of fractions through the concepts of division of whole numbers and multiplication of fractions.

In Primary Three, students have learnt the basic concepts of fraction. In this learning unit, students are required to recognise that fractions can be regarded as the quotients or the ratios of two whole numbers. However, the symbol of ratio ":" needs not be introduced.

Students should perform division of fractions and mixed arithmetic operations of fractions, both include those involving whole numbers, such as the division of a fraction by a whole number and the division of a whole number by a fraction. Teachers should let students recognise that the conventional order of performing mixed arithmetic operations of fractions is the same as that of whole numbers. In this learning unit, the division of fractions involves not more than three numbers while the mixed arithmetic operations of fractions involve three numbers only. Division of three fractions and mixed operations of multiplication and division of three fractions should not involve more than one mixed number.

Students should solve problems of division of fractions, including those situations involving remainders. For example, "Each pen costs $14\frac{2}{5}$ dollars, how many pens can be bought with 60 dollars? How much is left?"

Students have learnt to solve problems involving direct proportion by the unitary method in the Learning Unit 4N5 "Four arithmetic operations (II)". In this learning unit, the learning content on this topic will be further extended to deal with cases that involve the operations of

non-integers. For example, “72 dollars can buy 10 ball pens, how much do 8 ball pens cost?” Students are not required to use the term “direct proportion”.

Teachers should avoid requiring students to perform complicated division and mixed arithmetic operations of fractions. In this learning unit, if the result of calculations is a fraction, it should be reduced to the lowest terms. If it is greater than 1, it may be expressed as a fraction in the lowest terms, either as a mixed number or an improper fraction.

Problems involving finding the original numbers, such as the following problems, are tackled in Learning Units 5A2 “Simple equations (I)” and 6A1 “Simple equations (II)”:

- if $\frac{1}{3}$ of a number is 30, find this number
- if $\frac{2}{3}$ of a number is 30, find this number

Problems involving finding the fraction of a number by which it is greater or less than another number, and finding the fractional change of a number when it changes to another number are not required at the primary level. For example:

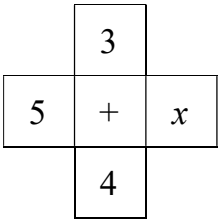
- by what fraction of 80 is 100 greater than it
- by what fraction of 100 is 80 less than it
- what is the fractional increase when 100 is increased to 120
- what is the fractional decrease when 120 is decreased to 100

After students have mastered the skills of performing division of fractions and mixed arithmetic operations of fractions, teachers should guide students to estimate the results of calculations and let them understand the advantages of estimation, so as to enhance students’ abilities in solving related problems.

Learning Unit	Learning Objective	Time
Algebra Strand		
5A1 Elementary algebra	1. recognise the use of letters to represent numbers 2. use algebraic expressions to represent the operations of and relations between quantities that are described in words and involve unknown quantities	6

Explanatory Notes:

This learning unit is the first one of the Algebra Strand at the primary level. Teachers may help students to recognise the use of letters to represent numbers through various learning activities, e.g. in the diagram below that if the sum of two numbers in the vertical line is equal to that in the horizontal line, what is the value of x?



Teachers may guide students to use algebraic expressions to represent the operations of and relations between quantities that are described in words and involve unknown quantities through different examples, e.g. using algebraic expression to represent "the sum of 8 and 2 times of a". The algebraic expressions in this learning unit should involve only one unknown quantity.

Besides, students are required to recognise the representations, such as:

- $3x$ is $3 \times x$, $x \times 3$ or $x + x + x$
- $\frac{x}{3}$ is $x \div 3$, $\frac{1}{3} \times x$ or $x \times \frac{1}{3}$

Teachers may discuss the advantages of using algebraic expression to represent the relations between quantities through different examples, e.g. "John is 2 years older than his sister. What are their ages?" If algebraic expressions are used, the relation between the ages of John and his sister can be clearly shown.

Examples of vocabularies to be learnt: algebraic expression, unknown quantity, etc.

Learning Unit	Learning Objective	Time
Algebra Strand		
5A2 Simple equations (I)	1. recognise the concept of equations 2. solve simple equations 3. solve problems by using equations	8

Explanatory Notes:

Students have already acquired a preliminary recognition to algebraic expressions at this stage. Teachers may guide students to recognise the concept of equations through various contexts. For example, teachers may ask students to solve the following problem "I have 15 dollars. How much more money should I add to get 50 dollars?" Teachers may set up an equation accordingly through which teachers could introduce the concepts of equations and the equity of the values on both sides of an equation.

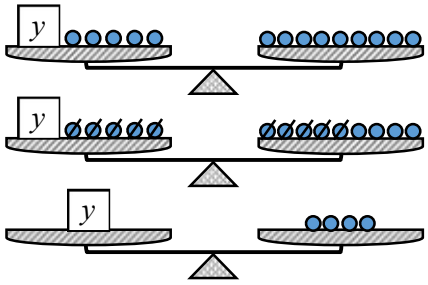
In Learning Objective 2, the types of simple equations that students are required to solve include:

1. $x + b = c$ 2. $x - b = c$ 3. $ax = b$ 4. $\frac{x}{a} = b$

5. $ax + b = c$ 6. $ax - b = c$ 7. $\frac{x}{a} + b = c$ 8. $\frac{x}{a} - b = c$

where a, b and c are whole numbers, and a is nonzero.

In Learning Objective 2, teachers should use balance to model and explain the process of solving an equation, such as



$y + 5 = 9$

$y + 5 - 5 = 9 - 5$

$y = 4$

Students are required to solve problems by using equations. Teachers should guide students to set equations directly according to the contexts of the problems, so as to avoid student setting the equations such as $y = 9 - 5$.

In this learning unit, students are required to recognise how to check the answers after solving equations or problems.

Examples of vocabularies to be learnt: equation, solving equation, solution of equation, etc.

Learning Unit	Learning Objective	Time
Measures Strand		
5M1 Area (II)	1. recognise the concept of height of triangles and quadrilaterals 2. recognise and use the formulae for finding the areas of parallelograms, triangles and trapeziums 3. find the areas of polygons	8.5

Explanatory Notes:

Students have learnt the concept of area and the formulae for areas of squares and rectangles in Primary Four. In this learning unit, teachers may guide students to apply their prerequisite knowledge to investigate the formulae for areas of parallelograms, triangles and trapeziums. Students should also use the formulae for areas that they have learnt to find the areas of polygons.

Learning Objective 1 aims at broadening student's recognition on the height of 2-D shapes. Students have learnt "given a straight line and a point not on the straight line, draw a straight line through that point and perpendicular to the given straight line" in Primary Two. Based on students' prerequisite knowledge, teachers may guide students to recognise the concept of height of triangles and quadrilaterals with different examples. Students are only required to recognise the corresponding height of each side of triangles and convex quadrilaterals. However, they are not required to use the term "convex quadrilaterals".

In Learning Objective 2, teachers may guide students to recognise the formulae for finding the areas of parallelograms, triangles and trapeziums through inquiry activities, such as dissecting and forming shapes. Students are required to use the formulae for finding the areas of parallelograms, triangles and trapeziums to solve problems. Problems on finding the lengths of sides of a triangle or a trapezium from its area should be dealt with in Learning Unit 6A1 "Simple equations (II)".

In Learning Objective 3, teachers may guide students to make use of the formulae for areas of 2-D shapes that they have learnt to find the areas of polygons in different ways, such as dissecting a polygon or making a new shape from the given one plus additional 2-D shape(s). Although the operations may involve more than five numbers in this learning unit, teachers

should avoid selecting and designing problems with complicated calculations for students to solve.

Examples of vocabularies to be learnt: base, height, polygon, etc.

Learning Unit	Learning Objective	Time
Measures Strand		
5M2 Volume (I)	1. recognise the concept of volume 2. compare intuitively the volumes of objects 3. recognise cubic centimetre (cm3) 4. measure and compare the volumes of objects in cubic centimeter 5. recognise cubic metre (m3) 6. recognise and use the formulae for finding the volumes of cubes and cuboids	7

Explanatory Notes:

In this learning unit, students have to recognise the concept of volume. Teachers may use learning activities together with daily life examples to guide students to recognise the concept, such as discussion on the amount of space that occupied by different types of objects in a box. In Learning Objective 2, students are required to compare the volumes of objects intuitively. Teachers should select two objects which are significantly different in volume to enable students to compare their volumes by observation. Teachers may help students understand the need for using standard units through activities.

After introducing cubic centimetre (cm^3) and cubic metre (m^3) as units of volume, teachers may help students recognise cubic centimetre and cubic metre through various learning activities. Students are required to measure and compare the volumes of objects in cubic centimetre. However, interconversion between cubic centimetre and cubic metre is not required. After students have acquired the experience of making measurements, teachers should teach them how to estimate the results of measurements and encourage them to estimate the result before measuring.

In Learning Objective 6, teachers may guide students to recognise formulae for finding the volumes of cubes and cuboids through inquiry activities, such as counting the number of cubes contained in a cuboid. Students are required to use the formulae for finding the volumes of cubes and cuboids to solve problems. However, finding the length of a cube from its volume is not required.

Students are required to find the volumes of simple 3-D shapes formed by cubes and cuboids. Although operations of the related problems may involve more than five numbers, teachers should avoid selecting and designing problems involving complicated calculations for students to solve.

Examples of vocabularies to be learnt: volume, cubic centimetre (cm^3), cubic metre (m^3), etc.

Learning Unit	Learning Objective	Time
Shape and Space Strand		
5S1 Circles	1. recognise the concept and basic properties of circles 2. draw circles	2.5

Explanatory Notes:

Students are required to recognise the concept and the basic properties of circles, which include the concepts of centre, radius, diameter and circumference of a circle.

The basic properties of circles include:

- all the points on a circle are at equidistant from its centre
- with line segments joining any two end points on a circle, those passing through the centre are the longest
- the length of the diameter is twice of the length of the radius

Teachers may let students use different methods to draw circles, including using compasses. Students should draw circles of specific radii or diameters. Teachers may let students freely create their own drawings which are formed by circles, and let them appreciate the beauty of geometric shapes.

The concept of circles is a prerequisite knowledge for learning the concept and properties of spheres. This learning unit should be taught prior to Learning Unit 5S2 “3-D shapes (III)”.

Examples of vocabularies to be learnt: centre, radius, diameter, circumference, compass, etc.

Learning Unit	Learning Objective	Time
Shape and Space Strand		
5S2 3-D shapes (III)	1. recognise the cross sections of prisms and cylinders 2. recognise the cross-sections of pyramids and cones 3. recognise the concepts of vertices and edges of a 3-D shape 4. recognise the concepts of cubes and cuboids 5. recognise the nets of cylinders 6. recognise the concept and basic properties of spheres	11

Explanatory Notes:

Students have learnt the concepts of the bases and lateral faces of prisms and pyramids, the bases and curved surfaces of cylinders and cones, and the curved surfaces of spheres in Primary Two. In this learning unit, students are required to recognise the concept of cross sections of 3-D shapes. They should recognise that the sizes and shapes of the cross sections of the prisms and the cylinders that are parallel to the bases are the same as that of the bases while the sizes of the cross sections of pyramids and cones that are parallel to their bases are different from that of the bases. Students are not required to use the term “uniform cross sections” to describe cross sections of prisms and cylinders.

Students are required to recognise the concepts of the vertices and edges of 3-D shapes and recognise the concepts of cubes and cuboids in quadrilateral prisms. At primary level, teachers should use cuboids that are not cubes as examples to explain the concepts of cuboids. Teachers should let students participate in activities of making frameworks of cubes and cuboids to enhance their recognition of the concepts of vertices, edges and faces of 3-D shapes. The construction of frameworks of prisms and pyramids is not a must. The relations among the number of edges, the number of vertices and the number of faces of 3-D shapes are not required in the curriculum. Teachers may deal with this content in Enrichment Topic 5E2 “Exploration of 3-D shapes”.

Students are required to recognise the nets of cubes, cuboids and cylinders. They should make the nets of cubes and cuboids.

Teachers should let students understand the concept and the basic properties of spheres through the concept of circles. The basic properties of spheres include:

- all the points on the sphere are at equidistant from the centre
- all the cross sections of a sphere are circles

Teachers may make use of concrete objects or computer software to help students recognise the basic properties of spheres. Students are not required to recognise the concept of greatest sectional circles.

Examples of vocabularies to be learnt: net, vertex, edge, cross section, centre, cube, cuboid, etc.

Learning Unit	Learning Objective	Time
Data Handling Strand		
5D1 Bar charts (III)	1. recognise compound bar charts 2. interpret compound bar charts 3. construct compound bar charts	6

Explanatory Notes:

Students have recognised bar charts in Primary Three and Primary Four. Teachers may make use of students' prerequisite knowledge to guide them recognising the limitation of bar charts. For example, we cannot find the ball game that the boys like most from the bar chart showing the favourite ball games of all students in the school. Then, teachers may guide students to recognise the advantage of using compound bar charts to present data.

In this learning unit, students are only required to interpret and construct compound bar charts using the one-to-thousand, one-to-ten thousand or one-to-hundred thousand representations. Compound bar charts in horizontal and vertical forms are required.

Students may start to write multi-digit numbers in ways such as 1 million, 2.5 million and 10 million in the strand of Data Handling, but they are required to avoid inappropriate style of writing such as 130 hundred and 3 hundred thousand.

Teachers may ask students to collect data prepared by government departments and related organisations from the media or the Internet, e.g. population by age group and gender. Then, teachers may let students construct compound bar charts after choosing the one-to-thousand, one-to-ten thousand or one-to-hundred thousand representations according to the magnitude of the data and round off the data to fit the chosen representation. Apart from paper and pencil, teachers may let students use IT to construct compound bar charts.

Examples of vocabularies to be learnt: compound bar chart, etc.

Learning Unit	Learning Objective	Time
Further Learning Unit		
5F1 Inquiry and investigation	Through various inquiry activities and mathematical modelling learning activities, construct and apply mathematical knowledge, further improve the ability to inquire, communicate, reason, conceptualise mathematical concepts and apply mathematics	10

Explanatory Notes:

This Learning Unit aims at providing students with more opportunities to engage in the activities that avail themselves of discovering and constructing knowledge, further improving their abilities to inquire, communicate, reason and conceptualise mathematical concepts when studying other Learning Units; as well as enhancing their abilities to apply mathematics to solve real-world problems through mathematical modelling activities. In other words, this is not an independent and isolated learning unit. The time is allocated for students to engage in learning activities from different learning units, for example, activities on enrichment topics, cross learning unit activities, and cross-KLA activities that based on mathematical topics. Among the learning activities, teachers should plan about 2–3 hours of mathematical modelling learning activities for students in each school year, enabling them to analyse problems and make decisions with mathematical thinking when encountering problems in different real-world contexts. and strengthening their ability to apply mathematics to solve STEAM and other real-world problems. Teachers may make good use of Mathematics lessons, lessons designated for project learning or cross-disciplinary learning, or other suitable lessons, together with other learning time, such as after-school or post-examination activity periods, to conduct mathematical modelling learning activities. These learning activities should be regarded as STEAM learning activities of Mathematics.

For details of the learning and teaching of mathematical modelling, please refer to pages 63 –72.

Learning Unit	Learning Objective	Time
Number Strand		
6N1 Decimals (IV)	1. perform division of a number by 10, 100, 1000 2. perform division of a number by 0.1, 0.01, 0.001 3. perform the division involving decimal(s) 4. perform mixed arithmetic operations of not more than four numbers 5. solve problems	9.5

Explanatory Notes:

Teachers should let students understand the concept of division of decimals through the concept of division of whole numbers.

Teachers should provide students with concrete examples to enable them to discover that dividing a whole number or a decimal by 10, 100 and 1000, are equivalent to moving the decimal point to the left by one, two, and three digits respectively; while dividing a whole number or a decimal by 0.1, 0.01 and 0.001, are equivalent to moving the decimal point to the right by one, two and three digits respectively.

Students should perform the following division involving decimal(s):

- decimal ÷ whole number
- whole number ÷ whole number（the quotient is a decimal）
- whole number ÷ decimal
- decimal ÷ decimal

The numbers of digits involved in the division of decimals, except the division in Learning Objectives 1 and 2, should not exceed that involved in the division of whole numbers as stated in Learning Unit 4N2 “Division (II)” (i.e. after adjusting the divisor into a whole number if needed, the dividend involves at most 3 digits and the divisor involves at most 2 digits). For example, the following divisions are not required:

- 12.34 ÷ 5.6 (=123.4 ÷ 56)
- 12.3 ÷ 5.67 (=1230 ÷ 567)
- 123 ÷ 0.4 (=1230 ÷ 4)

Students should perform mixed arithmetic operations up to four decimals. Teachers should let students recognise that the conventional order of performing mixed operations of decimals is the same as that of the whole numbers. Requirements for the numbers of digits involved in the multiplication and division of decimals are the same as those in the Learning Units 5N4 "Decimals (III)" and this learning unit.

Students should round off the result of calculations to the nearest tenth or hundredth when necessary, and use the symbol "≈" to indicate approximately equal.

Students should solve problems of division of decimals, including those situations involving remainders. For example, "Each pen costs 14.8 dollars, how many pens can be bought with 60 dollars? How much is left?"

After students have mastered the skills of performing division of decimals and mixed arithmetic operations of decimals, teachers should guide students to estimate the results of calculations and let them understand the advantages of estimation, so as to enhance students' abilities in solving related problems.

Learning Unit	Learning Objective	Time
Number Strand		
6N2 Decimals (V)	1. perform the interconversion between a decimal and a fraction 2. compare the magnitude of fractions by converting them into decimals	4.5

Explanatory Notes:

Students should convert decimals into fractions with denominators 10, 100, or 1000 and express the results in the lowest terms. For example, $0.7 = \frac{7}{10}$, $5.024 = 5\frac{24}{1000} = 5\frac{3}{125}$, etc.

In Primary Five, students learnt the relation between fractions and division. Teachers should let students recognise that dividing the numerator by the denominator of a fraction is the common way of converting the fraction into a decimal, but some fractions can also be converted into decimals by means of expansion. For example, $\frac{3}{25} = \frac{12}{100} = 0.12$.

Students should round off the result of calculations to the nearest tenth or hundredth when necessary. For example, when the quotient has too many decimal places or is a recurring decimal. They are also required to recognise how to estimate the results of calculations. However, teachers do not need to introduce the term “recurring decimal” and the concept of recurring decimals is not required in the primary Mathematics curriculum.

Students should use different methods to compare the magnitudes of fractions, including by converting fractions into decimals to compare their magnitudes.

Learning Unit	Learning Objective	Time
Number Strand		
6N3 Percentages (I)	1. recognise the concept of percentages 2. perform the interconversion between a percentage and a decimal 3. perform the interconversion between a percentage and a fraction	7

Explanatory Notes:

Teachers should use daily-life examples to let students recognise the concept of percentages. For example, 20% of students in the school wear glasses, the bus fare will increase by 5% next year, etc.

Students are required to recognise that a percentage represents the relation of the two quantities of the same kind and is expressed as a fraction with a denominator of 100.

Students are required to recognise that 100%=1 and 100% represents the whole. They should perform the interconversion between a percentage and a decimal, and perform the interconversion between a percentage and a fraction using the concepts of decimals, fractions and percentages. When converting a percentage into a fraction, students should express the result in the lowest terms. For example, 2.5% = $\frac{1}{40}$, $\frac{4}{5}\%$ = $\frac{1}{125}$, etc.

Teachers could guide students to discover some short cuts for interconversion between a percentage and a decimal, e.g., moving decimal points for the interconversions: 0.99 = 99%, 122% = 1.22, etc.

Examples of vocabularies to be learnt: percentage, etc.

Learning Unit	Learning Objective	Time
Number Strand		
6N4 Percentages (II)	1. solve problems	7

Explanatory Notes:

Students should solve simple problems related to percentages and percentage changes, such as:

- what percentage of 50 is 30
- what is 60% of 50
- what is the result when 50 is increased by 10%
- what is the result when 50 is decreased by 10%

Problems involving discount, interest or complicated situation related to percentages and percentage changes are not required. For example:

- what percentage is 100 more than 80
- what percentage is 80 less than 100
- what is the percentage increase from 100 to 120
- what is the percentage decrease from 120 to 100

Problems involving finding the original numbers, such as the following problem, are tackled in Learning Unit 6A1 "Simple equations (II)":

- if 75% of a number is 30, find this number

After students have mastered the calculation skills involving percentage, teachers should guide students to estimate the results of calculations and let them understand the advantages of estimation, so as to enhance students' abilities in solving related problems.

Learning Unit	Learning Objective	Time
Algebra Strand		
6A1 Simple equations (II)	1. solve simple equations involving non-integral coefficients or constants 2. solve problems by using equations	9

Explanatory Notes:

Students have learnt how to solve eight types of simple equations in Learning Unit 5A2 "Simple Equations (I)". In this learning unit, students further learn to solve simple equations involving non-integral coefficients or constants. The types of simple equations include:

1. $ax + b = c$
2. $ax - b = c$
3. $a(x + b) = c$
4. $a(x - b) = c$
5. $dx + ex = c$
6. $dx - ex = c \quad (d \neq e)$

where a, b and c can be whole numbers, fractions, decimals or percentages; whereas d and e must be whole numbers; and a, d and e are nonzero.

All types of equations in Learning Unit 5A2 "Simple Equations (I)" belong to either the first or the second types of equations mentioned above. When teaching students to solve simple equations involving non-integral coefficients or constants, equations such as $x + 2.5 = 3$, $x - 1\frac{1}{2} = 1.5$ and $\frac{2}{5}x = 40\%$ should be included. The like terms in the fifth and sixth types of equations should all be shown on only one side of the equations. Teachers should use balance or other tools to model and explain the process of solving an equation.

Students are required to solve problems by using equations. Teachers should guide students to set equations directly according to the contexts of the problems, so as to avoid student setting equations such as $y = 10 \div 2 - 3$.

In Learning Objective 2, students are required to use equations to solve problems in other learning units, such as:

- problems on finding the lengths of sides of a 2-D shape from its perimeter or area
- problems on finding the original values by its given percentages or fractions

In this learning unit, students are required to recognise how to check the answers after solving an equation or a problem.

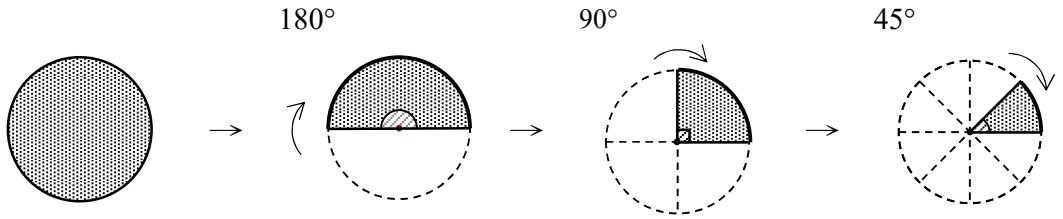
Examples of vocabularies to be learnt: equation, etc.

Learning Unit	Learning Objective	Time
Measures Strand		
6M1 Angle (degree)	1. recognise degree (°) 2. measure and compare the sizes of angles in degree 3. draw angles of given sizes	4

Explanatory Notes:

Students have learnt the concepts of angle, right angle, acute angle and obtuse angle in Primary Two. In this learning unit, teachers may help students understand the need for using standard units through appropriate learning activities, such as describing two acute angles of different sizes.

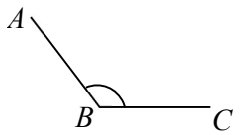
After introducing degree (°), teachers may help students further recognise degree through various activities, such as paper folding.



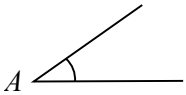
A round craft paper → Fold paper in half → Fold it in half again → Fold it in half for the third time

Students are required to measure and compare the sizes of angles in degree. Students are required to measure angles within 360° using protractors (0° and 360° are not required). After students have acquired the experience of making measurements, teachers should teach them how to estimate the results of measurements and encourage them to estimate the result before measuring.

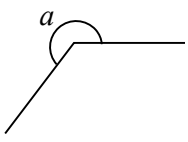
Students are required to recognise reflex angles, straight angles and round angles. Students are also required to name angles with the symbol "$\angle$", such as $\angle A$ and $\angle ABC$.



This is $\angle ABC$ (or $\angle CBA$)



This is $\angle A$



This is $\angle a$

Students are required to draw angles of given sizes by using protractors. They are also required to draw angles within 360° (0° and 360° are not required).

Examples of vocabularies to be learnt: degree (°), reflex angle, straight angle, round angle, clockwise, anti-clockwise, etc.

Learning Unit	Learning Objective	Time
Measures Strand		
6M2 Volume (II)	1. recognise the relation between capacity and volume 2. find the volumes of irregular solids by the water displacement method	8

Explanatory Notes:

Students have learnt the concepts of capacity and volume in Primary Three and Primary Five respectively. They have also learnt how to solve problems by using the formulae for finding the volumes of cubes and cuboids. In this learning unit, teachers may guide students to recognise the relation between capacity and volume through appropriate learning activities, such as recognising the relation between litre, millilitre and cubic centimetre by using a 1-litre measuring cup, a cubic box of side 10 cm and some cubes of side 1 cm. Teachers may guide students to recognise 1 cm^3 is equivalent to 1 mL and 1 m^3 is equivalent to 1000 L, etc. Students are required to recognise that m^3, cm^3, L and mL are units of volume of liquid, as well as units of capacity.

Teachers may guide students to find the volumes of irregular solids by the water displacement method through learning activities. Students are required to find the volumes of irregular solids by using tanks, measuring cups and overflow vessels. Teachers may guide students to recognise the points to note when finding the volumes of irregular solids by the water displacement method. For examples, the solids must be covered by water completely and water displacement method is not applicable if the solids float on water or melt in water.

Although operations may involve more than five numbers in this learning unit, teachers should avoid selecting and designing problems involving complicated calculations for students to solve.

Examples of vocabularies to be learnt: water displacement method, irregular solid, etc.

Learning Unit	Learning Objective	Time
Measures Strand		
6M3 Perimeter (II)	1. recognise pi 2. recognise and use the formula for circumference	3.5

Explanatory Notes:

Students have learnt the concept of perimeter in Primary Four. In this learning unit, teachers may guide students to investigate the relation between circumference and diameter through appropriate learning activities, such as measuring and comparing diameters and circumferences of different circles, and guide them to recognise pi. Student are required to recognise that pi can be represented by " π " and some approximate values of π.

Teachers may make use of the learning activities on recognition of pi to guide students to recognise the formula for circumference. Students are required to use the formula for circumference to solve problems. The following types of problems can be included

- finding the perimeters of circles, semi-circles, quarter-circles and 2-D shapes formed by them
- finding the diameter and radius of a circle from its circumference

However, students are not required to learn the formula for finding the length of an arc given its angle subtended at the centre. Related content belongs to the junior secondary Mathematics curriculum.

In this learning unit, students are only required to use $\frac{22}{7}$ or 3.14 as approximate values of π for calculations. Requirements for the numbers of digits involved in the multiplication and division of decimals respectively in Learning Objectives 5N4.3 and 6N1.3 are not applicable to this learning unit. However, calculations of numbers with too many digits should be avoided. In this learning unit, operations may involve more than five numbers but teachers should avoid selecting and designing problems involving complicated calculations for students to solve.

Examples of vocabularies to be learnt: circumference, pi, π, etc.

Learning Unit	Learning Objective	Time
Measures Strand		
6M4 Speed	1. perform the interconversion between units of time 2. solve problems related to time intervals 3. recognise the concept of speed 4. compare intuitively the speed of objects 5. compare directly the speed of objects 6. compare the speed of objects in improvised units 7. recognise metres per second (m/s) and kilometres per hour (km/h) 8. interpret travel graphs 9. solve problems related to speed	10

Explanatory Notes:

Students have learnt the relations among hours, minutes and seconds at Key Stage One. They have also learnt the concepts of fractions, decimals and the related operations. In Learning Objective 1, the interconversion between units of time should involve single units only. Students are only required to interconvert between hour and minute, and between minute and second, such as 90 minutes = 1.5 hours / $1\frac{1}{2}$ hours, 180 seconds = 3 minutes.

In Learning Objective 2, students are required to solve problems related to time intervals. Given any two of the starting time, finishing time and time interval, students are required to find the unknown quantity/time. Both hours and minutes can be involved in the time intervals. However, teachers should avoid selecting and designing problems with complicated calculations for students to solve. Problems on calculations involving hours, minutes and seconds at the same time are not required. For example, John has spent 1 hour and 15 minutes, 24 minutes and 30 seconds, and 20 minutes and 30 seconds to do physical exercises in three different time slots today. How much time does he spend on doing physical exercise today?

Teachers may guide students to recognise the concept of speed in some contexts that students are familiar with, such as running competitions.

In Learning Objective 4, students are required to compare the speed of objects intuitively. Teachers should select two objects which are significantly different in speed to enable students to compare their speeds by observation. Teachers may let students to compare the speed of objects directly through activities and then discuss with them how to compare the speed of objects in improvised units, such as measuring the distances travelled by a model car in a designated time before and after its modification. At the end of activities, teachers may guide students to understand the need for using standard units.

After introducing the units of speed metres per second (m/s) and kilometres per hour (km/h), teachers may enhance student's recognition on metres per second and kilometres per hour through learning activities, such as discussing the speeds of motor-cars and aeroplanes. However, interconversion between metres per second and kilometres per hour is not required. Students are required to solve problems related to speed. However, problems on chasing are not required.

Teachers may guide students to recognise travel graphs through appropriate learning activities, such as storytelling, and help them to solve problems related to speed through interpreting travel graphs.

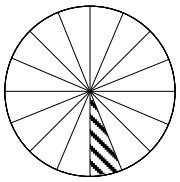
Although operations may involve more than five numbers in this learning unit, teachers should avoid selecting and designing problems with complicated calculations for students to solve.

Examples of vocabularies to be learnt: speed, metres per second (m/s), kilometres per hour (km/h), travel graph, etc.

Learning Unit	Learning Objective	Time
Measures Strand		
6M5 Area (III)	1. recognise the formula for areas of circles 2. use the formula for finding the areas of circles	3

Explanatory Notes:

Students have learnt the concept of area and the formulae for areas of square, rectangle, parallelogram, triangle and trapezium in Primary Four and Primary Five. In this learning unit, teachers should guide students to recognise the formula for area of circle through appropriate learning activities, such as folding a circular piece of paper into 4 equal parts and further folding it into 8, 16 … equal parts. Students can see that each part looks like a triangle. Students can then explore the formula for areas of circles by regarding those equal parts as triangles



Teachers may encourage students to recognise the stories of ancient Chinese mathematicians (such as *Liu Hui* (劉徽) and *Zu Chongzhi* (or *Tsu Chung-chi*, 祖沖之)) on finding the value of pi. Emphasis is placed on the contributions of Chinese mathematicians on finding the value of pi, but not on the explanation on the methods of calculations.

Students are required to use the formula for finding the areas of circles to solve problems. However, students are not required to:

- find the diameter or radius of a circle from its area
- find the area of a sector by using $\frac{\text{angle subtended at the centre}}{360°} \times$ area of circle.

Teachers may guide students to make use of their prerequisite knowledge on fractions to find areas of shapes such as quarter-circle, semi-circle and three quarter-circle.

In this learning unit, students are only required to use $\frac{22}{7}$ or 3.14 as approximate values of π for calculations. Requirements for the numbers of digits involved in the multiplication and

division of decimals respectively in Learning Objectives 5N4.3 and 6N1.3 are not applicable to this learning unit. However, calculations of numbers with too many digits should be avoided. In this learning unit, operations may involve more than five numbers but teachers should avoid selecting and designing problems involving complicated calculations for students to solve.

Examples of vocabularies to be learnt: area of circle, etc.

Learning Unit	Learning Objective	Time
Shape and Space Strand		
6S1 Symmetry	1. recognise the concept 2-D shapes having axial symmetry 2. draw and make 2-D shapes having axial symmetry	4.5

Explanatory Notes:

Students are required to recognise the concept of 2-D shapes having axial symmetry and find the axes of symmetry of these shapes. Teachers should let students fold a 2-D shape having axial symmetry along its axis of symmetry to enable students to recognise the meaning of complete coincidence of the two sides.

Students are required to recognise that squares, rectangles, isosceles triangles, equilateral triangles, rhombuses and circles are 2-D shapes having axial symmetry and recognise their axes of symmetry. Although some teachers make use of real-life axial symmetry shapes patterned with figures to introduce the concept of axial symmetry, students are only required to determine whether a 2-D shape is axial symmetry, the curriculum does not require students to consider the figures on the shape.

Students are required to draw and make 2-D shapes having axial symmetry, e.g. to draw 2-D shapes having axial symmetry on grid papers and to make 2-D shapes having axial symmetry by paper cutting. When given a half of a 2-D shape being divided according to its axis of symmetry, students should draw the other half along the axis of symmetry.

Examples of vocabularies to be learnt: 2-D shape having axial symmetry, axis of symmetry, etc.

Learning Unit	Learning Objective	Time
Data Handling Strand		
6D1 Averages	1. recognise the concept of averages 2. find the average of a group of data 3. solve problems	3.5

Explanatory Notes:

In this learning unit, teachers should explain the concept of averages by using some statistical charts, e.g. pictograms or bar charts of smaller frequency counts.

Apart from finding the average of a group of data by calculations, students are required to recognise how to estimate the results of calculations and the average of the group of data from statistical charts. For example, teachers may let students draw a line in a bar chart to show the estimated value of average. Then, teachers may ask students to move the parts of the bars above the line to try filling the bars below the line, for helping them to give the final estimation of average.

Regarding the limitations of representing a group of data by its average, such as the effect of extreme values in a group of data to its average, would be introduced at junior secondary. Teachers may select some contexts that students are familiar with when asking them to solve problems involving averages. Teachers may ask students to find a missing datum of a group of data based on its average for enhancing students' recognition to the relation between a group of data and its average.

Operations in this learning unit may involve more than five numbers.

Examples of vocabularies to be learnt: average, etc.

Learning Unit	Learning Objective	Time
Data Handling Strand		
6D2 Broken line graphs	1. recognise broken line graphs 2. interpret broken line graphs 3. construct broken line graphs	4.5

Explanatory Notes:

Students have learnt bar charts and compound bar charts from Primary Three to Primary Five. In this learning unit, teachers may use bar charts on continuous data for guiding students to recognise broken line graphs, e.g. the height of a student in different years.

Teachers may let students recognise the similarities and differences between broken line graphs and bar charts through discussion. For example, although the amount of each item can be effectively compared in both types of graphs, the change of data is easier to be observed from broken line graphs.

In this learning unit, problems involving complicated calculations should be avoid although there is no limitations on the scales adopted by the broken line graphs to be interpreted and constructed by students. Moreover, it is suggested that there should not be more than two sets of data being presented by broken lines on the same broken line graph.

Teachers may ask students to collect some appropriate information to construct broken line graphs, e.g. daily maximum or minimum temperature in a week. Teachers may remind students the points to note when constructing broken line graphs. For example, the scale of vertical axis may affect the perception of readers to the changes of data.

Apart from using paper and pencil, teachers may let students use IT to construct broken line graphs of greater frequency counts.

Teachers may consider using real-life examples or related learning elements in Science Education or Technology Education KLAs to enhance learning and teaching. For example, teachers may ask students to search for the monthly temperature of Hong Kong and a city in Australia from the Internet and study that the four seasons appear in different months in the two places.

Examples of vocabularies to be learnt: broken line graph, tendency, etc.

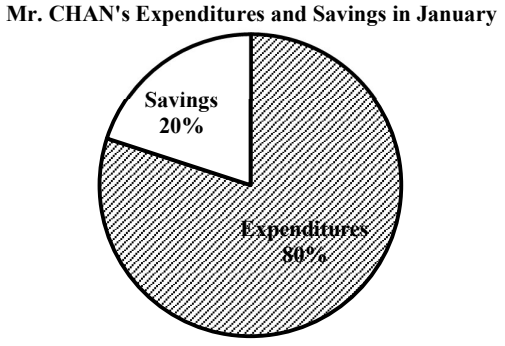
Learning Unit	Learning Objective	Time
Data Handling Strand		
6D3 Pie charts	1. recognise pie charts 2. interpret pie charts	4

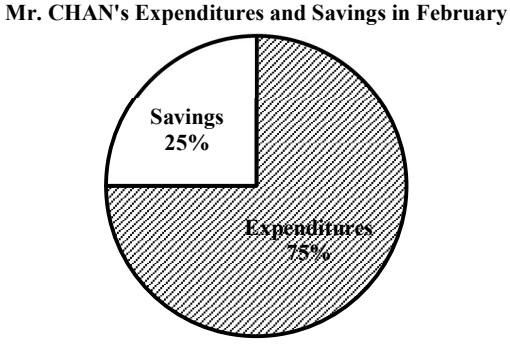
Explanatory Notes:

Students have learnt bar charts in Primary Three and Primary Four. In this learning unit, teachers may make use of bar charts, e.g. the time that a student spending on different activities in one day, to guide students to recognise pie charts. Teachers may also help students to understand what information of the data could be more effectively shown in a pie chart. For example, the proportion of each item to the whole.

In this learning unit, students are only required to interpret pie charts involving simple calculations. If the pie charts are presented with the angles at the centre, the angle at the centre of each sector should be a multiple of 30° or 45°. Students are not required to measure the angles at the centre of a pie chart for calculations. Moreover, students are only required to perform simple calculation to interpret pie charts labelled with percentages.

Teachers may use examples to remind students the points to note when interpreting pie charts. For example, we cannot determine whether the total amount of Mr. CHAN's saving in January is fewer than that in February from the following pie charts.





In this learning unit, students are not required to construct pie charts with paper and pencil. However, teachers may consider to let students construct pie charts using IT.

Examples of vocabularies to be learnt: pie chart, angle at the centre, etc.

Learning Unit	Learning Objective	Time
Data Handling Strand		
6D4 Uses and abuses of statistics	1. present the data with appropriate statistical charts 2. discuss and recognise the uses and abuses of statistical charts in daily life	3

Explanatory Notes:

For learning objective 1, students should use the statistical knowledge that they have learnt to solve problems, e.g. collecting and organising data effectively, choosing appropriate statistical charts to present data and interpreting the statistical charts appropriately. Teachers may help students to revise the organisation and representation of data, and discuss with students on choosing appropriate statistical charts from pictograms, bar charts, broken line graphs and pie charts for presenting data through examples.

For learning objective 2, teachers may help students to discuss and recognise the uses and abuses of statistical charts in daily life through examples. For example, if we present a group of data by some broken line graphs with vertical axes of different scales, readers' perception to the change of a quantity may be different. Teachers should help develop in students the attitude of handling data with rigor and the adverse effects of presenting data inaccurately, such as misleading readers and directing people to make wrong decisions.

Examples of vocabularies to be learnt: uses and abuses of statistical charts, etc.

Learning Unit	Learning Objective	Time
Further Learning Unit		
6F1 Inquiry and investigation	Through various inquiry activities and mathematical modelling learning activities, construct and apply mathematical knowledge, further improve the ability to inquire, communicate, reason, conceptualise mathematical concepts and apply mathematics	10

Explanatory Notes:

This Learning Unit aims at providing students with more opportunities to engage in the activities that avail themselves of discovering and constructing knowledge, further improving their abilities to inquire, communicate, reason and conceptualise mathematical concepts when studying other Learning Units; as well as enhancing their abilities to apply mathematics to solve real-world problems through mathematical modelling activities. In other words, this is not an independent and isolated learning unit. The time is allocated for students to engage in learning activities from different learning units, for example, activities on enrichment topics, cross learning unit activities, and cross-KLA activities that based on mathematical topics. Among the learning activities, teachers should plan about 2–3 hours of mathematical modelling learning activities for students in each school year, enabling them to analyse problems and make decisions with mathematical thinking when encountering problems in different real-world contexts. and strengthening their ability to apply mathematics to solve STEAM and other real-world problems. Teachers may make good use of Mathematics lessons, lessons designated for project learning or cross-disciplinary learning, or other suitable lessons, together with other learning time, such as after-school or post-examination activity periods, to conduct mathematical modelling learning activities. These learning activities should be regarded as STEAM learning activities of Mathematics.

1. The Process of Mathematical Modelling

Mathematical modelling is the process of applying mathematical knowledge and tools to solve real-world problems. It emphasises on thinking about problems and analysing data in a mathematical way. Teachers should guide students to **analyse the real-world problem**: clarify key factors of the problem, identify related information/data and the underlying mathematical elements, simplify the problem by putting forward assumptions, convert it into a mathematical problem, and determine relations between relevant quantities and variables. Students apply their prior mathematical knowledge and skills to **formulate a mathematical model** and **solve it for a mathematical solution**, and then **interpret the mathematical solution** by considering the meaning of the solution in the context of the real-world problem to obtain a **real-world solution**. Subsequently, students **evaluate the mathematical model** through validation and reflection to suggest refinements of the model. Figure 1 and Table 1 on the next page show the five steps of the mathematical modelling process and the relevant elements of each step respectively:

Figure 1: The process of mathematical modelling

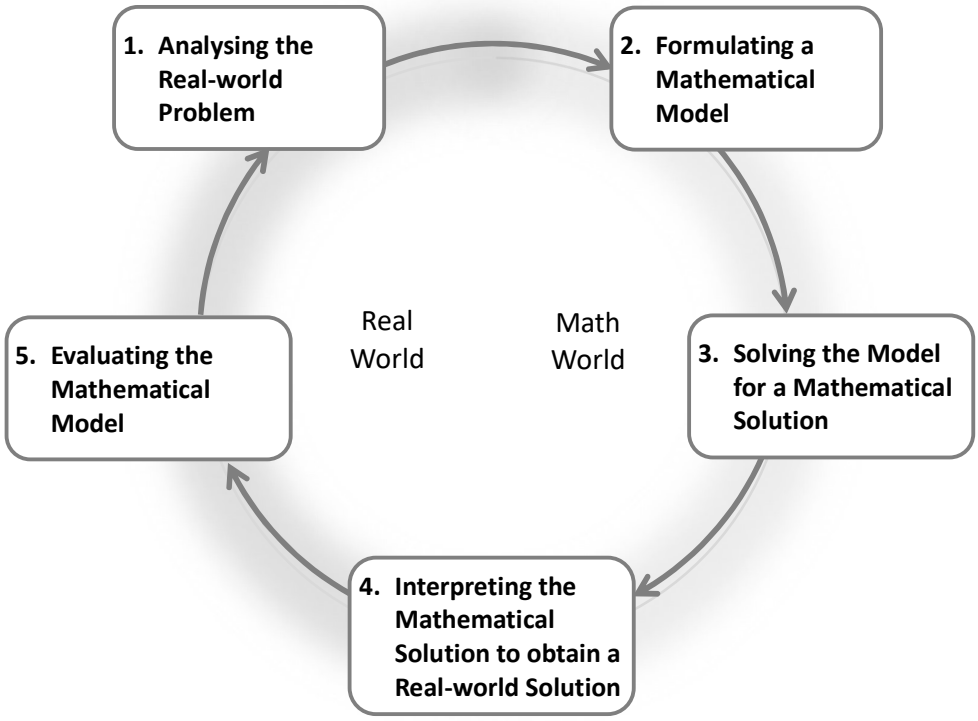


Table 1: Elements of each step in Mathematical Modelling

Steps	Elements
1. Analysing the Real-world Problem	• Understand the real-world context • Clarify key factors of the problem • Identify related information/data and mathematical elements of the problem
2. Formulating a Mathematical Model	• Put forward assumptions to simplify the real-world problem • Convey the problem in a mathematical way • Determine relations between key quantities/variables
3. Solving the Model for a Mathematical Solution	• Apply mathematical knowledge, skills and different tools to solve the model
4. Interpreting the Mathematical Solution to Obtain a Real-world Solution	• Consider the meaning of the mathematical solution in the context of the real-world problem
5. Evaluating the Mathematical Model	• Validate the model against the real-world context • Reflect on the strengths and limitations of the model • Compare different models • Suggest refinements of the model

The steps and elements of the mathematical modelling process shown on the previous page aim to provide teachers with a general reference for designing mathematical modelling learning activities. Teachers may adjust individual steps or omit certain elements to suit the actual teaching circumstances and students' abilities. For example, if an activity does not involve multiple mathematical models, the element "Compare different models" in Step 5 "Evaluating the Mathematical Model" can be omitted.

At Key Stage 2, mathematical modelling learning activities focus on providing students with the experience of one complete mathematical modelling process. Students should realise that mathematics can be used to model real-world situations for solving problems. At this key stage, after building a model, students are not required to go through the modelling process again to refine the model. Teachers may also introduce the modelling steps briefly, without asking them to identify or memorise the modelling steps, nor to explain or reflect on their own thinking processes according to each modelling step.

Regarding the interface between the learning of mathematical modelling across key stages at the primary level, students at Key Stage 1 may lay a good foundation for learning mathematical modelling at Key Stage 2 through handling word problems set in daily life contexts by employing counting, classifying and comparing to express and analysis problems mathematical, as well as through learning mathematics in different strands. Teachers could consider not to arrange mathematical modelling learning activities for students at Key Stage 1.

To suit students' learning abilities, mathematical modelling learning activities at Key Stage 2 should focus on the first four steps of the modelling process, namely "**Analysing the Real-world Problem**", "**Formulating a Mathematical Model**", "**Solving the Model for a Mathematical Solution**" and "**Interpreting the Mathematical Solution to Obtain a Real-world Solution**". As for Step 5 "Evaluating the Mathematical Model", it can be handled in a simpler way. Students will learn the whole mathematical modelling process in greater depth at later key stages. For the learning of mathematical modelling at different key stages, please refer to Table 3 "Notes on the Learning and Teaching of Mathematical Modelling across Key Stages".

2. Design of Mathematical Modelling Learning Activities

Problems in mathematical modelling learning activities are different from conventional mathematical word problems. Contexts of word problems are usually simplified. The conditions and information needed for solving the problems are given, complete and just sufficient. They are mainly designed to consolidate students' understanding of mathematical concepts and their application of specific skills. In contrast, problems in mathematical modelling learning activities are generally unsimplified real-world problems. They are open-ended and do not lead to a specified or explicit solving approach, nor a unique or optimal solution. Furthermore, the relevant conditions and information may not be fully provided. Students need to make suitable assumptions to simplify the problem, as well as collect and select the necessary information/data, to facilitate the formulation of a mathematical model that fits the context.

As students learn mathematical modelling for the first time at this key stage, teachers should note the following when designing mathematical modelling learning activities:

- Teachers should choose **daily life contexts that students are familiar with** as modelling topics, such as shopping, travelling and school life, to suit students' mathematical abilities;
- Teachers should provide students with the **information/data** needed for the activities (e.g. charts taken from relevant websites). Teachers should help students formulate **simple mathematical models** through close guidance;
- The mathematical content involved in the activities may cover any knowledge that students have learnt. It **need not be restricted to the grade level of students**. For example, teachers may design a mathematical modelling learning activity for Primary 4 students that involves the Primary 3 Learning Unit "3M5 Weight";
- Depending on the modelling activities, teachers may consider allowing students to use calculators or mathematical software applications to reduce their time on calculations;
- Besides designing activities that cover **all the mathematical modelling steps**, teachers may also design activities that focus on individual steps when students first learn mathematical modelling. This helps students get to know the modelling process step by step. For example, for an activity that covers the first four steps, students may spend more time on the first two steps to build simple models, and then they may complete the remaining two steps briefly;
- Teachers may refer to the curriculum documents of other subjects, such as primary Science, to design **cross-disciplinary** mathematical modelling learning activities. This helps students integrate and apply knowledge from different subjects to solve problems;
- Teachers may design mathematical modelling learning activities in different forms, such as classroom activities or project learning, to suit students' abilities and needs;
- Teachers may include elements of **values education** in suitable mathematical modelling learning activities, based on the real-world contexts of the activities;
- Teachers may **use artificial intelligence (AI) to assist in designing activities, organising data, predicting students' learning difficulties, producing related learning resources**, etc. This helps cater for learner diversity and raise students' interest and engagement. Teachers **must carefully review the output of AI** to ensure the quality of learning and teaching, as well as students' learning effectiveness.

3. Learning and Teaching of Mathematical Modelling

At this key stage, teachers should offer close guidance and support to help students go through the five steps of mathematical modelling, with a focus on the first four steps. To help students experience and begin to learn the mathematical modelling process, teachers should note the following in learning and teaching:

<u>Step 1: Analysing the Real-world Problem</u>

- Teachers should provide well-structured mathematical modelling learning activities and the information students need. For example, reading materials of specific or organised information are provided to students to helps them acquire basic understanding of the real-world context of the problem. Teachers should also guide students to think about which factors are more important for solving the problem and are related to mathematics, so as to assist them to clarify the key factors, related information/data and mathematical elements of the problem, and prepare them to formulate the mathematical model;
- For more able students, teachers may guide them to collect useful information for solving the problem by themselves.

Step 2: Formulating a Mathematical Model

- Teachers should help students understand that real-world problems are usually complicated. Students need to make assumptions to simplify the real-world problem when formulating a mathematical model;
- Teachers may give students some examples of assumptions and guide them to make their own assumptions through discussion. Teachers can then guide students to express the problem in a mathematical way. For example, a problem about "feasible routes" can be turned into a mathematical problem about key factors that can be measured, compared or calculated, such as distance and fare;
- Teachers may use numerical expressions, manipulatives, diagrams or tables to help students understand and determine the relations between key quantities for formulating the mathematical model;
- Depending on the problem context and teaching needs, teachers may let students make different assumptions and build the corresponding mathematical models.

Step 3: Solving the Model for a Mathematical Solution

- Depending on the problem context and data, teachers may guide students to focus on one mathematical model, or to discuss different mathematical models in groups;
- The solving process should focus on developing students' ability to apply their prior mathematical knowledge and skills flexibly, and to use tools appropriately;
- Teachers may guide students to use different digital tools, such as calculators, spreadsheets and drawing tools, to help solve the model. This reduces tedious written calculations and drawing work. For example, when the model involves mixed operations of decimals, students may use spreadsheets to do the calculations, so that teachers and students can spend more time on discussing the ways to solve the model.

Step 4: Interpreting the Mathematical Solution to Obtain a Real-world Solution

- Teachers should guide students through questioning or discussion to consider the meaning of model solutions in the context of the real-world problem. For example, if the real-world problem is to find the number of vehicles needed and the solution of the model is not a whole number, students need to adjust it to obtain the real-world solution.

Step 5: Evaluating the Mathematical Model

- Teachers should guide students to validate the model against the real-world context, for example, by comparing the results of the model against actual observations. Students should also reflect on the strengths and limitations of the model. If the problem is suitable, teachers may further guide students to suggest refinements to the model;
- Teachers could consider not to ask students to formulate a refined model based on the refinements they suggest.

When students answer teachers' questions, discuss in groups or report on their discussions, teachers may encourage them to express their own views for building an open and rational atmosphere for discussion. Teachers should avoid rushing to fixed answers, models or mathematically better methods, so as to helps develop students' critical thinking and creativity.

If **AI is to be applied in activities**, teachers **should first let students think independently or understand the methods for handling the data. Teachers can then work alongside students and guide them to use AI to obtain feedback on their thinking, or to help with tedious calculations, etc.** Teachers must let students understand that AI is only used for supporting their inquiry and reflection, and that AI-generated content may not be correct or appropriate. Students need to **carefully review the output of AI** and **avoid letting AI think for them**. This builds their awareness of using AI responsibly.

4. Assessment of Mathematical Modelling

Regarding the assessment of mathematical modelling, teachers should take note of the following:

- Teachers can assess students' performance in each modelling step by reviewing both the process and outcomes of the modelling process, such as what reflected in classroom discussions, oral presentations, worksheets, etc. Teachers can set appropriate assessment criteria to help understand students' learning outcomes in different steps of the modelling process, for providing appropriate feedback;
- For different mathematical modelling learning activities, teachers may use assessment forms to help assess students' performance in both process and outcomes. An assessment form may be designed based on the modelling steps and related elements involved in the modelling activity. Teachers can then give comments and suggestions for improvement based on students' performance in each step and in the whole activity. For example, regarding a primary 4 mathematical modelling learning activity focusing on the first four steps of the modelling process, teachers may design an assessment form that matches the learning objectives of this activity (see Table 2 on the next page);
- Teachers can provide timely oral or written feedback at different stages of mathematical modelling learning activities. Peer assessment can also be conducted to help students understand their performance and improve their learning. Teachers can also use the assessment results to improve the delivery of mathematical modelling learning activities.

5. Learning and Teaching Resources

Regarding the examples of mathematical modelling learning activities, please refer to the learning and teaching resources on the website of the Mathematics Education Section of Education Bureau.

Table 2: Example of an Assessment Form

<table>
<tr><th>Activity</th><th>Elements*</th><th>Evaluation / Feedback</th></tr>
<tr><td>Activity 1</td><td>- Able to clarify key factors of the problem
- Able to identify related information/data and mathematical elements of the problem</td><td rowspan="3">(Teachers can provide comments on student performance at different stages of the activity based on the nature of the activity and the mathematical modelling elements involved. They can also give marks or grades accompanied by corresponding descriptions.)</td></tr>
<tr><td>Activity 2</td><td>- Able to put forward/select appropriate assumptions to simplify the real-world problem
- Able to convey the problem in an appropriate mathematical way</td></tr>
<tr><td>Activity 3</td><td>- Able to apply mathematical knowledge, skills and different tools to solve the model
- Able to consider the meaning of the mathematical solution with respect to the real-world problem</td></tr>
<tr><td colspan="3">Overall feedback:</td></tr>
</table>

* The elements in the table could be adapted to suit the mathematical modelling learning activities by adding, removing or further specifying individual elements. For example, if students need to collect information in Activity 1, the element "Able to collect relevant information" could be added to Activity 1. If Activity 3 only involves obtaining results with the four arithmetic operations, its element "Able to apply mathematical knowledge, skills and different tools to solve the model" may be changed to "Able to obtain results by applying four arithmetic operations".

Table 3: Notes on the Learning and Teaching of Mathematical Modelling across Key Stages

		Key Stage 2 (P4 – P6)	Key Stage 3 (S1 – S3)	Key Stage 4 (S4 – S6)
Problem	1. Nature and complexity of problems	• Based on daily life contexts that students are familiar with • Related information/data are given • Guide students to formulate a simple model	• Based on real-life contexts that students are familiar with • The given information/data can be surplus or insufficient • Require students to formulate one or several models	• Based on real-life contexts that student may or may not be familiar with • The given information/data can be unstructured or scattered in diverse data sources • Require student to formulate several models or a series of models
Teacher	2. Teacher instruction	• Provide well-structured mathematical modelling learning activities along with clear and comprehensive instructions, as well as close guidance and support	• Provide sufficient guidance and support while allowing students to have the space to explore the problem themselves	• Facilitate students to conduct mathematical modelling activities themselves by providing timely guidance and support throughout the process
Student	3. Students' recognition of modelling process	• Be guided to experience the mathematical modelling process, with relatively less emphasis on model evaluation • Be aware that mathematics can be used to model real-world scenarios for solving problems, while not required to identify nor memorise modelling steps	• Be guided to recognise the modelling process according to the five steps of the modelling process	• Learn to conduct mathematical modelling activities themselves according to the five steps of the modelling process

		Key Stage 2 (P4 – P6)	Key Stage 3 (S1 – S3)	Key Stage 4 (S4 – S6)
	4. Students' modelling capability	• Put forward or recognise the assumptions to simplify the real-world problem	• Make reasonable assumptions to simplify the real-world problem	• Make reasonable assumptions to simplify the real-world problem and be able to provide an explanation on the relevance and reasonableness
		• Express real-world problems as simple mathematical problems and formulate related mathematical models after considering the key quantities/variables provided	• Express the real-life problem as a mathematical problem and formulate the related model(s) through identifying key quantities/variables and represent the relationship between them mathematically	• Express the more complicated real-life problem as a mathematical problem and formulate the related models through defining, identifying and organising key quantities/variables mathematically
		• Put forward or recognise possible strengths and limitations of the models	• Perform evaluation of model, such as comparing the results with real-world data, to identify the strengths and limitations of the model	• Understand the importance of model evaluation and perform evaluation of the model to identify the strengths and limitations of the model
			• Propose possible refinements to the model	• Make refinements to the model
	5. Students' application of mathematical knowledge and tools during the modelling process	• Apply mathematical knowledge and skills of lower and upper primary, such as those about arithmetic operations, areas, percentages, averages, etc. in the modelling process	• Apply prior knowledge and mathematics knowledge acquired at junior secondary, such as linear equations, polynomial, trigonometry, probability, etc. in the modelling process	• Apply prior knowledge and mathematics knowledge acquired at senior secondary, such as functions and graphs, variations, 3-D geometry, probability, differentiation*, matrices* and probability distribution* in the modelling process

		Key Stage 2 (P4 – P6)	Key Stage 3 (S1 – S3)	Key Stage 4 (S4 – S6)
		• Use manipulatives, diagrams, tables and simple mathematical expressions to present the modelling results	• Use diagrams, tables, algebraic expressions and graphs to present the modelling results	• Use diagrams, tables, functions and their graphical representations to present the modelling results
		• Use calculators and simple digital tools, etc. as appropriate in the modelling process	• Use digital tools, such as spreadsheet, graphing software, dynamic geometry software and artificial intelligence tools, etc. as appropriate in the modelling process	• Use digital tools, such as spreadsheet, graphing software, dynamic geometry software, calculus calculator*, matrix calculator* and artificial intelligence tools, etc. as appropriate in the modelling process

* Applicable to students taking Module 1 or Module 2.

Acknowledgements

We would like to thank the members of the following Committees for their invaluable comments and suggestions in the compilation of this booklet.

CDC Committee on Mathematics Education

Ad Hoc Committee on Primary Mathematics Curriculum

Ad Hoc Committee for Integrating Mathematical Modelling into Primary Mathematics Curriculum

